**Lecture 2.**

**Some Set Theory. Heine-Borel Theorem.**

**Bolzano-Weierstrass Theorem.**

**Some Set Theory.**

**Definition.** *contains* and we write , if every member of is also a member of . In this case, is a subset of .

**Definition.** A *equals* B , and we write , if A contains B and B contains A; thus, if and only if A and B have the same members, i.e.

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**Definition.** The *complement of* A, denoted by  , is the set of elements in the universal set that are not in A.

**Definition.** The *union* of A and B , denoted by **, is the set of elements that are at least in A or B, i.e.



**Definition.** The *intersection of sets* A and B , denoted by **, is the set of elements that are in both A and B, i.e.

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Properties:

,





**Definition.**  is the set of elements that are in A but not in B, i.e.



**Definition.** The *symmetric difference* of sets A and B, denoted by

∆, is the set of elements that are in union of sets  and , i.e.

**∆** or ∆

**Supremum of set.**

Let *А* be a set of real numbers. A set *A* of real numbers is bounded above if there is a real number  such that  whenever . In this case,  is an upper bound of *A*.

If  is an upper bound of *A*, then so is any larger number. If  is an upper bound of *A*, but no number less than  is, then  is a supremum of *A*, and we write

= sup *A.*

In other words  is a supremum of a set *A* if :

1.  is an upper bound of *A*, i.e. for each  following inequality  holds.
2.  is the smallest number of all upper bounds, number  less than  can’t be the upper bound of a set *A.*



**Definition.**

1. : 



2. , : .

**Infimum of set**

A set *A* of real numbers is bounded below if there is a real number such that  whenever . In this case, is a lower bound of *A*. If is a lower bound of *A*, so is any smaller number. If is a lower bound of *A*, but no number greater than is, then is an infimum of *A*, and we write

In other words is an infimum of a set *A* if :

1.  is a lower bound of *A*, i.e. , for each  the following inequality  holds.
2.  is the greatest number of all lower bounds, number  more than  can’t be the lower bound of a set *A.*

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**Definition.**

1. : 



2. , : 

**Theorem . (Heine-Borel Theorem)** If H is an open covering of a closed and bounded subset S of the real line, then S has an open covering consisting of finitely many open sets belonging to H.

**The Bolzano-Weierstrass Theorem**

As an application of the Heine-Borel theorem, we prove the following theorem of Bolzano and Weierstrass.

**Theorem . (Bolzano-Weierstrass Theorem)** Every bounded infinite set of real numbers has at least one limit point.